Appendix 1

Creation of a Standing Wave inside a Reflective Sphere.

To derive the equations in three dimensions the equations consider the reflections in a Spherical system.

Three-Dimensional Model  A hollow sphere is mirrored on the inner surface and has a coherent light source at its center. The light from the source is reflected back by the mirror creating a set of concentric spherical standing waves.

As the sphere is symmetrical about the X axis, we can derive the equations in two dimensions and rotate the result through 180 degrees to get the results for 3 dimensions. Removing the sides of the sphere allows us to see the effects in the resulting ring. The R axis describes an axis orthogonal to the X axis.

As the sphere moves, the emitted waveform is compressed in the direction of travel, and is stretched in the opposite direction. Figs 2.

To examine the path of the light inside the sphere we will consider what happens to a ray of light emitted at an angle \( \theta \) down a tube of length \( 2R \), which passes through the center of the sphere. The light source is at the mid point of a tube and the tube has mirrors mounted at each end, and they are perpendicular to its axis.

A pulse the light from the source will travel down the length of the tube until it strikes the mirrors, and then it will be reflected back to the source. As the mirrors are at the surface of the sphere, this is the same as if we sent a single beam of light to the shell of the sphere and it is reflected back to the source.

In both cases the light reflects back along the same path, and thus this ray of light has been isolated from the others. In both cases the light returns to the source.

In the next case the sphere and the tube are moving, but for the occupants of the sphere the results are the same.

However for stationary observers their observations show that the light expands in rings as shown in figure 3. The light has a wavelength of \( \lambda \) and the circles, representing the beginning of a cycle, are centered on the point of emission. In the tube the light passes along the tube’s axis and returns along its axis to the center, which is also the center of the sphere.

The light ring which passes through points A and B was emitted 1 time cycle earlier at time \( t = -1 \). The center and the source have now, at \( t = 0 \), reached point P. The light emitted at time \( t = 0 \) has not yet left P.

This is shown in figure 3 and is examined in figure 4.
Refering to the diagram below, in the reference system the light ray in the tube follows the path OA, but appears to travel from P to A in the moving system.

After the light has expanded for \( c/\lambda = t \) seconds (one cycle), the leading edge of the wave has reached point A in Figure 4. It has traveled distance \( OA = ct \) in space, but only distance PA in the tube. The wavelength of the light in the moving system, \( \lambda_T \), is therefore PA.

Using simple geometry and trigonometry, the equation for the wavelength of the light in the tube can be derived as a function of the angle \( \theta \), the angle between the axis of the tube and the X axis. The length PA (\( \lambda_T \)) is the wavelength of the light emitted at angle \( \theta \). The wavelength of the light at the complimentary angle, \( 180 - \theta \), is \( PB(\lambda_R) \).

Let \( \beta = v/c \).

\[
\begin{align*}
CP &= OP \cos(\theta) \\
&= vt \cos(\theta) \\
&= ct \left( s \cos(\theta) \right) \\
&= \lambda s \cos(\theta)
\end{align*}
\]

\[
\begin{align*}
CA &= \sqrt{(ct)^2 - (vt \sin(\theta))^2} \\
&= ct \left( \sqrt{1 - \beta^2 \sin^2(\theta)} \right) \\
&= \lambda \sqrt{1 - \beta^2 \sin^2(\theta)} \\
\lambda_A &= CA - CP \\
\lambda_A &= \lambda \left( \sqrt{1 - \beta^2 \sin^2(\theta)} - s \cos(\theta) \right)
\end{align*}
\]

Similarly

\[
\begin{align*}
\lambda_B &= CA + CP \\
\lambda_B &= \lambda \left( \sqrt{1 - \beta^2 \sin^2(\theta)} + \beta \cos(\theta) \right)
\end{align*}
\]

\( \lambda_B \) is the wavelength of the transmitted signal. To find the wavelength of the reflected signals we can replace the mirror on the end of the tube by a second identical source, moving at the same velocity, which is aligned so that the light is emitted back down the tube at an angle of \( 180 - \theta \). The result is that the reflected light has a wavelength of \( \lambda_R \).

Thus the transmitted and reflected signals have the following wavelengths and frequencies.

\[
\begin{align*}
\lambda &= cl \\
s &= v/c \\
CP &= v \cos(\theta) \\
CA &= cl \left( 1 - s \cdot \sin^2(\theta) \right) \\
CA &= BC \\
PA &= CA - CP = \lambda_1 \\
BP &= BC + CP = \lambda_2
\end{align*}
\]

Figure 3: The Doppler Shift in 2D. The expanding wave-fronts are emitted from the source P as it moves to the right.

Figure 4: Geometrical Calculation of the Doppler Wavelength in 2D, based on Figure
\[ \lambda_T = \lambda \left( \sqrt{(1 - \beta^2 \sin^2 \theta)} - \beta \cos \theta \right) \]
\[ \lambda_R = \lambda \left( \sqrt{(1 - \beta^2 \sin^2 \theta)} + \beta \cos \theta \right) \]
\[ f_T = \frac{c}{\lambda_T} \tag{3} \]
\[ f_R = \frac{c}{\lambda_R} \tag{4} \]

Expressed in radians/sec this is:
\[ \omega_R = 2\pi f_R \tag{5} \]
\[ \omega_T = 2\pi f_T \tag{6} \]

These result in the familiar Doppler equations in the
case where \( \theta = 0 \).
\[ \lambda_T = \lambda(1 - \beta) \]
\[ \lambda_R = \lambda(1 + \beta) \]